Logic: The Theory of Inquiry, 1938*

John Dewey†

Contents

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1 Introduction

1.1 The Problem of Logical Subject Matter

1. Logic is a progressive discipline

p. 14 .... As the methods of the sciences improve, corresponding changes take place in logic.

2. The subject-matter of logic is determined operationally

.... Operations ... fall into two general types. There are operations that are performed upon and with existential material – as in experimental observation. There are operations performed with and upon symbols. .... The former are performed upon existential conditions; the latter upon symbols. But the symbols in the latter case stand for possible final existential conditions while the conclusion, when it is stated in symbols, is a precondition of further operations that deal with existences. .... Operations involve both material and instrumentalities, including in the latter tools and techniques.

3. Logical forms are postulational

p. 16

Inquiry in order to be inquiry in the complete sense has to satisfy certain demands that are capable of formal statement .... According to the view that makes a basic difference between logic and methodology, the requirements in question subsist prior to and independent of inquiry. Upon that view, they are final in themselves, not intrinsically postulational. This conception of them is the ultimate ground of the idea that they are completely and inherently a priori and are disclosed to a faculty called pure reason. The position here taken holds that they are intrinsically postulates of and for inquiry, being formulations of conditions, discovered in the course of inquiry itself, which further inquiries must satisfy if they are to yield warranted assertibility as a consequence.

pure reason

logical forms are postulates for all inquiries

warranted assertibility

p. 17 ....

Just as the postulates of, say, geometry are not self-evident first truths that are externally imposed premises but are formulations of the conditions that have to be satisfied in procedures that deal with a certain subject-matter, so with logical forms which hold for every inquiry.

postulates of geometry

4. Logic is a naturalistic theory

p. 18

.... means, one one side, that there is no breach of continuity between operations of inquiry and biological operations and physical operations. "Continuity," on the other side, means that rational operations grow out of organic activities, without being identical with that from which they emerge. ....

The logic in question is also naturalistic in the sense of the observability, in the ordinary sense of the word, of activities of inquiry. Conceptions derived from a

mystical faculty of intuition or anything that is so occult as not to be open to public inspection and verification (such as the purely psychical for example) are excluded.

intuition, mystical faculty of

5. Logic is a social discipline

p. 19.

.... man is naturally a being that lives in association with others in communities possessing language, and therefore enjoying a transmitted culture. .... Those who are concerned with ”symbolic logic” do not always recognize the need for giving an account of the reference and function of symbols. While the relations of symbols to one another is important, symbols as such must be finally understood in terms of the function which symbolization serves.

symbolization, function of

p. 20

Language in its widest sense – that is, including all means of communication such as, for example, monuments, rituals, and formalized arts – is the medium in which culture exists and through which it is transmitted. .... Language is the record that perpetuates occurrences and renders them amenable to public consideration .... The naturalistic conception of logic ...is thus / cultural naturalism .

language

6. Logic is autonomous

.... it does not depend upon anything extraneous to inquiry. ...this proposition ...precludes resting logic upon metaphysical and epistemological assumptions and presuppositions.

1.2 The Existential Matrix of Inquiry: Biological

1.3 The existential Matrix of Inquiry: Cultural

p. 42

The environment in which human beings live, act and inquire, is not simply physical. It is cultural as well. Problems which induce inquiry grow out of the relations of fellow beings to one another, and the organs for dealing with these relations are not only the eye and ear, but the meanings which have developed in the course of living, together with the ways of forming and transmitting culture with all its constituents of tools, arts, institutions, traditions and customary beliefs.

p. 48

A sound or mark of any physical existence is a part of language only in virtue of its operational force ; that is, as it functions as a means of evoking different activities performed by different persons so as to produce consequences that are shared by all the participants in the conjoint undertaking. [cf. Wittgenstein, PU]

p. 49

Any word or phrase has the meaning which it has only as a member of a constellation of related meanings.

p. 50

The system [of symbol-meanings] may be simply the language in common use.

symbol meanings

Its meanings hang together not in virtue of their examined relationship to one another, but because they are current in the same set of group habits and expectation. They hang together because of group activities, group interests, customs and institutions. Scientific language, on the other hand, is subject to a test over and above this criterion. Each meaning that enters into the language is expressly determined in its relation to other members of the language system. .... The resulting difference in the two types of language-meanings fundamentally fixes the difference between what is called common sense and what is called science.

scientific language

p. 51

signs vs. symbols

The representative capacity [of natural signs] is attributed to things in their connection with one another , not to marks whose meaning depends upon agreement in social use. .... I prefer to mark the difference by confining the application of sign to so-called ”natural signs” – employing symbol to designate ”artificial” signs.” .... I shall .. connect sign and significance , symbol and meaning, respectively, with each other, in order to have terms to designate two different kinds of representative capacity. .... the important consideration is that existent things, as signs, are evidence of the existence of something else, this something being at the time inferred rather than observed. But words, or symbols, provide no evidence of any existence. Yet what they lack in this capacity they make up for in creation of another dimension. They make possible ordered discourse or reasoning. For this may be carried on without any of the existences to which symbols apply being actually present: without, indeed, assurance that objects to which they apply anywhere actually exist, and, as in the case of mathematical discourse, without direct reference to existence at all. .... symbols introduce into inquiry a dimension different from that of existence.

p. 54

inference vs. implication

Just as the sign-significance relation defines inference, so the relation of meanings that constitutes propositions defines implication in discourse, it it satisfies the intellectual conditions for which it is instituted. .... the confusion, when inference is treated as identical with implication, has been a powerful agency in creating the doctrinal conception that logic is purely formal – for ...the relation of meanings (carried by symbols) to one another is, as such , independent of existential reference.

p. 55

relation

The word ”relation” is used to cover three very different matters which in the interest of a coherent logical doctrine must be discriminated. (1) Symbols are ”related” directly to one another ; (2) they are ”related” to existence by the mediating intervention of existential operations ; (3) existences are ”related” to existence by the evidential sign-signified function. ...I shall reserve the word relation to designate the kind of ”relation” which symbol-meanings bear to one another as symbol-meanings. I shall use the term reference to designate the kind of relation they sustain to existence ; and the words connection (and involvement) to designate that kind of relation sustained by things to one another in virtue of which inference is possible.

reference

p. 56

language, communication, cooperation

.... it is language, originating as a medium of communication in order to bring about deliberate cooperation and competition in conjoint activities, that has conferred upon existential things their signifying or evidential power.

p. 60

1.4 Common Sense and Scientific Inquiry

p. 62 [Common sense] designates the conceptions and beliefs that are currently accepted without question by a given group or by mankind in general. They are common in the sense of being widely, if not universally, accepted. They are sense, in the way in which we speak of the ”sense of a meeting” and in which we say things do or do not ”make sense.” They have something of the same ultimacy and immediacy for a group that ”sensation” and ”feeling” have for an individual in his contact with surrounding objects. It is a commonplace that every cultural group possesses a set of meanings which are so deeply embedded in its customs, occupations, traditions and ways of interpreting its physical environment and group-life, that they form the basic categories of the language-system by which details are interpreted.

common sense

p. 65 . . . common sense inquiries are concerned with qualitative matter. . .

. . . . both the history of science and the present state of science prove that the goal of the systematic relationship of facts and conceptions to one another is dependent upon elimination of the qualitative as such and upon reduction to non-qualitative formulation.

p. 67

In actual experience, there is never any such isolated singular object or event; an object or event is always a special part, phase, or aspect, of an environing experienced world – a situation . . . . There is always a field in which observation of this or that object or event occurs.

situation

p. 68

We live and act in connection with the existing environment, not in connection with isolated objects, even though a singular thing may be crucially significant in deciding how to respond to total environment.

p. 76 . . . . common sense is concerned with a field that is dominantly qualitative while science is compelled by its own problems and goals to state its subject-matter in terms of magnitude and other mathematical relations which are non-qualitative. . . . since common sense is concerned, directly and indirectly, with problems of use and enjoyment, it is inherently teleological. Science, on the other hand, has progressed by elimination of ”final causes” from every domain with which it is concerned, substituting measured correspondences of change.

common sense qualitative, science quantative, mathematical

. . .

. . . these differences are due to the fact that different types of problems demand different modes of inquiry for their solution, not to any ultimate division in existential subject-matter.

p. 77

Prescientific ideas and beliefs in morals and politics are . . . so deeply ingrained in tradition and habit and institutions, that the impact of scientific method is feared as something profoundly hostile to mankind’s dearest and deepest interests and values.

p. 78

. . . physical science has, in practical fact, liberated and vastly extended the range of ends open to common sense and has enormously increased the range and power of the means available for attaining them.

p. 79

The attainment of unified method means that the fundamental unity of the structure of inquiry in common sense and science be recognized, their difference being one in the problems with which they are directly concerned, not in their respective logics. . . .. In the main, we are asked to take our choice between the traditional logic, which was formulated not only long before the rise of science but when also the content and methods of science were in radical opposition to those of present science, and the new purely "symbolistic logic" that recognizes only mathematics, and even at that is not so much concerned with methods of mathematics as with linguistic formulations of its results. The logic of science is not only separated from common sense, but the best that can be done is to speak of logic and scientific method as two different and independent matters. Logic in being "purified" from all experiential taint has become so formalistic that it applies only to itself.

logic so pure that it refers only to itself

p. 81

1.5 The needed reform of logic

p. 89

. . . the fundamental difference between the Greek conception of Nature as it is expressed in Aristotelian cosmology, ontology and logic, and the modern conception as that has been determined in the scientific revolution. The most evident point of difference concerns the entirely different position given to the qualitative and the quantitative in their relations to one another. It is not merely that classic cosmology and science were constituted in terms of qualities, . . . but that all quantitative determinations were relegated to the state of accidents, so that apprehension of them had no scientific standing.

. . . .

p. 90

There was, therefore, on the basis of the Aristotelian theory of Nature and knowledge no point or purpose in making measurements except for lower "practical" ends. . . . . Measuring was useful to the artisan in dealing with physical things, but that very fact indicated the gulf which separated quantity and measuring from science and rationality.

p. 91

Taking both measurement and relations into account, it is not too much to say that what Greek science and logic rejected are now the head corner-stone of science – although not yet of the theory of logical forms.

p. 93

. . . . Cartesian algebraic geometry, that effected determination of all figures by formulae of generalized numerical coordinates was more than a new instrument of scientific analysis and record. It marked the beginning of the logical movement by which all mathematical propositions became formulae for dealing with possible objects, not descriptions of their existing properties – so that they are logically non-existential in their content, save when taken to prescribe operations of experimental observation.

Cartesian geometry

p. 94

The attempt to retain Aristotelian logical forms after their existential

foundations have been repudiated is the main source of existing confusion in logical theory. It is the ultimate reason why logical forms are treated as merely formal.

. . . .

The authors of the classic logic did not recognize that tools constitute a kind of language which is in more compelling connection with things of nature than are words, nor that the syntax of operations provides a model for the scheme of ordered knowledge more exacting than that of spoken and written language. Genuine scientific knowledge revived when inquiry adopted as part of its own procedure and for its own purpose the previously disregarded instrumentalities and procedures of productive workers. This adoption is the radical characteristic of the experimental method of science. The great role of mathematics in the conduct of science shows that discourse still has a fundamental role. But as far as existential knowledge is concerned, that role is now subordinate and not supreme.

tools as language

experimental method

2 The Structure of Inquiry and the Construction of Judgements

2.6 The Pattern of Inquiry

p. 102

In everyday living, men examine;. . . ; they infer and judge as ”naturally” as they reap and sow, produce and exchange commodities. As a mode of conduct, inquiry is as accessible to objective study as are these other modes of behavior. Because of the intimate and decisive way in which inquiry and its conclusions enter into the management of all affairs of life, not study of the latter is adequate save as it is noted how they are affected by the methods and instruments of inquiry that currently obtain.

p. 109

An idea is first of all an anticipation of something that may happen; it marks a possibility.

idea, anticipation

p. 110

Every idea originates as a suggestion, but not every suggestion is an idea. The suggestion becomes an idea when it is examined with reference to its functional fitness; its capacity as a means of resolving the given situation.

. . .

Because suggestions and ideas are of that which is not present in given existence, the meanings which they involve must be embodied in some symbol. Without some kind of symbol no idea; a meaning that is completely disembodied can not be entertained or used.

no symbol, no idea

2.7 The Construction of Judgement

p. 128

”Substance” represents therefore, a logical, not an ontological determination. . . . The condition – and the sole condition that has to be satisfied in order that there may be substantiality, is that certain qualifications hang together as dependable signs that

certain consequences will follow when certain interactions take place.

p. 129

An object . . . is a set of qualities treated as potentialities for specified existential consequences. . . . The greater the number of interactions, of operations, and of consequences, the more complex is the constitution of a given substantial object. object

. . . . Being a substantial object defines a specific function.

p. 130

Aristotle: "It is absurd to make that fact that the things of this earth change and never remain the same the basis of our judgments about the truth. For in pursuing the truth one must start from things that are always in the same state and never change." Aristotle

p. 131

The meanings which are suggested as possible solutions of a problem, which are then used to direct further operations of experimental observation, form the predicational content of judgments. . . . . the practice of scientific inquiry has provided the foundations for a correct logical interpretation. The conceptual and "rational" contents are hypotheses. In their more comprehensive forms they are theories.

p. 135

. . . any sentence isolated from place and function in inquiry is logically indeterminate.

p. 136

Blueprints and maps are propositions and they exemplify what it is to be propositional. . . . .

Like a chart, indeed, like any physical tool or physiological organ, a proposition must be defined by its function. Furthermore, there is the same sort of advantage in having conceptual frameworks manufactured and on hand in advance of actual occasions for their use, as there is in having tools ready instead of improvising them when need arises.

p. 138

Apart from the limits set by the problem in hand, there are no rules whatever for determining what may or should be predicated . . . . Anything is "essential" which is indispensable in a given inquiry and anything is "accidental" which is superfluous.

2.8 Immediate Knowledge: Understanding and Inference

p. 140

There is continuity in inquiry. The conclusions reached in one inquiry become means, material and procedural, of carrying on further inquiries. In the latter, the results of earlier inquiries are taken and used without being resubjected to examination . . . . This immediate use of objects known in consequence of previous mediation is readily confused with immediate knowledge.

2.9 Judgments of Practice: Evaluation

p. 159

2.10 Affirmation and Negation: Judgment as Requalification

p. 181

2.11 The Function of Propositions of Quantity in Judgement

p. 213 .... The use of linguistic symbols, of number-names, is the invention which permitted quantity and number to become objects of independent or mathematical investigation. For the relations of symbols to one another in a meaning-symbol system can be examined on their own account, independent of the relations existential objects and changes sustain to one another. number-names

p. 215

The negation of quality or indifference to it which is sometimes ascribed to quantity and number ... is not final but, on the contrary, positive means for controlled construction of new objects and institution of new qualities. ....–so science renders things qualitatively unlike (as sounds and colors, pressures, light and electricity) comparable with one another, in such ways that controlled interchanges are capable of being brought about.

p. 217

... space and time are in science not what we measure but are themselves results of measurements of objects and events, in the interest of objective determination of problematic situations. ... A unit of measurement is, when it is taken as a unit of measurement, discrete. But it is internally continuous .... What is taken as discrete in one functional use is used as continuous in resolution of another problem and conversely. space time

p. 218

Counting is as existential an operation as is whistling or singing. Calculations in scientific work may go on in the head as well as they may be written down on paper. But symbols as symbols do not have physical efficacy. They have to be existentially manipulated if calculation occurs. The habit of ruling out the existential acts of counting and calculation from the domain with which logic is concerned is simply another instance of the systematic neglect of operations, so characteristic of formalistic logic, a neglect which is due to the doctrine that propositions are merely enunciative or declarative of antecedent existence or subsistence. counting

2.12 Judgment as Spatial-Temporal Determination: Narration-Description

p. 231

Logical theory is concerned with the relation existing between evidential data as grounds and inferences drawn as conclusions, and with the methods by which the latter may be grounded. With respect to logical theory, there is no existential proposition which does not operate either (1) as material for locating and delimiting a problem; or (2) as serving to point to an inference that may be drawn with some degree of probability; or (3) as aiding to weigh the evidential value of some data; or (4) as supporting and testing some conclusion hypothetically made. At every point, exactly as in conducting any inquiry into contemporary physical conditions, there has

to be a search for relevant data; criteria for selection and rejection have to be formed as conceptual principles for estimating the weight and force of proposed data, and operations of ordering and arranging data which depend upon systematized conceptions have to be employed.

p. 264

2.14 Generic and Universal Propositions

universal propositions

Universal propositions are formulations of possible ways or modes of acting or operating. . the universal is stated as a relation of an antecedent if content and a consequent then clause.

p. 268

kinds

Every proposition that involves the conception of a kind is based upon a set of related traits or characteristics that are the necessary and sufficient conditions of describing a specified kind.

p. 270

. . . the traits which descriptively determine kinds are selected and ordered with reference to their function in promoting and controlling extensive inference. . . . .

We habitually employ qualities as signs although we do not habitually or ”naturally” investigate their qualifications to be so taken and used.

p. 271

In a universal proposition, possibility of a mode of operation is expressed in an if-then form. . . . . The relation is question is one of temporal priority and consequence.

3 Propositions and Terms

3.15 General Theory of Propositions

p. 284

logical positivism

In emphasizing the symbolic element, [logical positivism] brings propositions into connection with language generically; and language, while about things directly or indirectly, is acknowledged to be of another dimension than that which it is about. Moreover, formulation of logical subject-matter in terms of symbols tends to free theory from dependence upon an alleged subjective realm . . . . For symbols and language are objective events in human experience.

p. 285-6

logical formalism

. . . logical positivism as usually formulated is so under the influence of logical formalism, derived from analysis of mathematics, as to make an over-sharp distinction between matter and form, under the captions of ”meaning of words” and ”syntactical relations.” . . . . But the necessity for the distinction does not decide whether they are or are not independent of each other: –Whether they are or are not, for example, intrinsically related to each other in logical subject-matter and distinguishable only in theoretical analysis. . . . this fact but poses in a new way the old fundamental problem of the relation, or absence of relation, between matter and form, or meanings and syntax.

meaning vs. syntax

p. 287

. . . the position here taken, . . . is that inquiry is concerned with objective transformations of objective subject-matter; that such inquiry defines the only sense in which "throught" is relevant to logic; and that propositions are products of provisional appraisals, evaluations, of existences and of conceptions as means of institution of final judgment which is objective resolution of a problematic situation. Accordingly, propositions are symbolizations, while symbolization is neither an external garb nor yet something complete and final in itself . . . . According to the position here taken, propositions are to be differentiated and identified on the ground of the function of their contents as *means*, procedural and material, further distinctions of forms of propositions being instituted on the ground of the special ways in which their respective characteristic subject-matters function as means . . . . But at this point it is pertinent to note that, since means as such are neither true nor false, truth-falsity is *not* a property of propositions. Means are either effective or ineffective; pertinent or irrelevant; wasteful or economical, the criterion for the difference being found in the consequences with which they are connected as means. On this basis, special propositions are *valid* (strong, effective) or *invalid* (weak, inadequate); loose or rigorous, etc.

situation, problematic

propositions as means

propositions valid / invalid, not true / false

3.16 Propositions ordered in Sets and Series

p. 311

3.17 Formal Functions and Cannons

p. 328

i. Formal Relations of Terms p. 336

ii. Formal Relations of Propositions p. 343

iii. Formal Cannons of Relations of Propositions p. 344

1. *Identity* . . .

p. 345

. . . . In scientific inquiry, every conclusion reached, whether of fact or conception, is held subject to determination by its fate in further inquires. Stability or 'ìdentity',' of meanings is a limiting ideal, as a condition to be progressively satisfied. The conditional status of scientific conclusions (conditional in the sense of subjection to revision in further inquiry) is sometimes used by critics to disparage scientific "truths" in comparison with those which are alleged to be eternal and immutable. In fact, it is a necessary condition of continuous advance in apprehension and in understanding.[6]

identity

scientific "truths"

2. *Contradiction*.

Peirce' definition of 'truth'

[6]The best definition of *truth* which is known to me is that of Peirce: "The opinion which is fated to be ultimately agreed to by all who investigate is what we mean by the truth, and the object represented by this opinion is the real." [[Pei34]], p. 268. A more complete (and more suggestive) statement is the following: "Truth is that concordance of an abstract statement with the ideal limit towards which endless investigation would tend to bring scientific belief, which concordance the abstract statement may possess by virtue of the confession of its inaccuracy and one-sidedness, and this confession is an essential ingredient of truth." (*Ibid.*, pp. 394-5).

. . . . For establishment of propositions one of which must be valid if the other is invalid is an indispensable step in arriving at a grounded conclusion . . . . Complete exclusion, resulting in grounded disjunction, is not effected until propisitions are determined as pairs such that if one is valid the other is invalid, and if one is invalid the other is valid. The principle of contradiction thus represents a condition to be satisfied. Direct inspection of two propositions does not determine whether or not they are related as contradictories, as would be the case of contradiction were an inherent relational property.

p. 346

e. *Excluded Middle* . . . . The principle of excluded middle presents the completely generalized formulation of conjunctive-disjunctive functions in their conjugate relation. The notion that propositions are or can be, in and of themselves, such that the principle of excluded middle directly applies is probably the source of more fallacious reasoning in philosophical discourse and in moral and social inquiries than any other one sort of fallacy . . . . To determine subject-matters so that no alternative is possible is the most difficult task of inquiry.

contradiction

p. 347.

A conclusion in mathematical discourse is as universal (since it is an abstract hypothetical proposition) as are those from which it follows.

3.18 Terms or Meanings

p. 349

4 The Logic of Scientific Method

4.19 Logic and Natural Science: Form and Matter

p. 371

. . . .

The intrinsic place of form in logical subject-matter is more than a commonplace. It states the character which marks off logical subject-matter from that of other sciences.

excluded middle

p. 391

. . . scientific method both constitutes and discloses the nature of logical forms . . . .

1. The history of actual scientific advance is marked by the adoption and invention of material devices and related techniques – of complex and refined forms of apparatus and definite related techniques of using apparatus . . . .

2. The new data thus instituted do much more than provide facts for confirming and refining old conceptions. They institute a new order of problems whose solution requires a new frame of conceptual reference . . . .

3. Upon the conceptual side, this scientific revolution was accompanied by a revolution in mathematical conceptions; again, partly as cause and partly as effect. As long as Euclidian geometry was taken to be the exemplary model of mathematical

form and matter

evolution in mathematical conceptions

method, the underlying categories of mathematics were such as to be applicable only to structures fixed within certain limits .... Cartesian analytics, th calculus, and subsequent developments were called for by the radically new emphasis placed in scientific inquiry upon correlations of change, while independent development of mathematical conceptions disclosed in their application to existence new, more refined and extensive problems of correlated change.

4.20 Mathematical Discourse

p. 394

The ability of any logical theory to account for the distinguishing logical characteristics of mathematical conceptions and relations is a stringent test of its claims. A theory such as the one presented in this treatise is especially bound to meet and pass this test. ...the interpretation of the logical conditions of mathematical conceptions and relations must be such as to account for the form of discourse which is intrinsically free from the *necessity* of existential reference while at the same time it provids the*possibility* of indefinitely extensive existential reference – such as is exemplified in mathematical physics.

Euclidian geometry vs. Cartesian analytics

mathematics and logic

I. *Transformation as a Fundamental Category.*

p. 395

The logical principle involved may be restated in the following ways: (1) The subject-matter or *content* of discourse consists of *possibilities* .... (2) As possibilities, they require formulation in symbols. Symbolization is not a convenience found to be practically indispensable in discourse, nor yet a mere external garb for ideas already complete in themselves. It is of the very essence of discourse as concerned with possibilities, in their functional capacity, however, symbols have the same logical status as existential data .... Historically, the operations by which symbol-meanings are transformed were first borrowed from and closely allied to physical operations – as in indicated in the words still used to designate rational operations; in gross, in such words as *deliberation, pondering, reflection,* and more specifically in *counting* and *calculation*. As meanings were modified to satisfy the conditions imposed by membership in an interrelated system, operations were also modified to meet the requirements of the new conceptual material. Operations became as abstract as the materials to which they apply and hence of a character expressed, and capable only of expression, in a new order of symbols.

existential reference in mathematics

p. 396

.... When ...discourse is conducted exclsively with reference to satisfaction of its *own* logical conditions, or, as we say, for its own sake, the subject-matter is not only non-existential in immediate reference but is itself formed on the ground of freedom from existential reference of even the most indirect, delayed and ulterior kind. It is then mathematical. The subject-matter is completely abstract and formal because of its complete freedom from the conditions imposed upon conceptual material which is framed with reference to final existential application. Commplete freedom and complete abstractness are here synonymous terms.

Change in the *context* of inquiry effects a change in its intent and contents. .... A ...new context is provided when all reference to existential applicability is eliminated. The result is not simply a higher degree of abstractness, but a new order of

abstractions, one that is instituted and controlled only by the category of abstract relationship. The necessity of transformation of meanings in discourse in order to determine warranted existential propositions provides, nevetheless, the connecting link of mathematics with the general pattern of inquiry.

p. 397

symbolization in mathematics

. . . music did not create in either nature or in speech sounds and their ordered arrangement. Music, however, developed the potentialities of sounds and their cadenced arrangement in activities having their own distinctive subject-matter. An analogy with development of mathematics is not forced. Numerical determinations first arose as means of economic and effective adjustment of material means to material consequences in qualitative situations marked by deficiency and excess. But not only was there nothing in the operations that were involved to obstruct development on their own account, but they invited such development.

music
mathematics and music

. . . Greek mathematicians and philosophers effected a partial liberation from existential reference. But abstraction was not complete. Conceptions of arithmetic and geometry . . . were supposed to refer to the metes and bounds existing in nature itself by which nature was an intelligible structure and by which limits were set to change. . . . The story of liberation of mathematical subject-matter from any kind of ontological reference is one with the story of its logical development through a series of crises, such as were presented by irrationals, negatives, imaginaries, etc.

p. 398-9

mathematics and economy

. . . . . The contents of a mathematical proposition, *qua* mathematical . . . have no meaning or interpretation save that wchich is formally imposed by the need of satisfying the condition of transformability within the system, with no extra-systemic reference whatever. In the sense which "meaning" bears in any conception having even indirect existential reference, the terms have no meaning – a fact which accounts, probably, for the view that mathematical subject matter is simply a string of arbitrary marks. But in the wider logical sense, they have a meaning constituted exclusively and wholly by their relations to one another as determined by satisfaction of the condition of transformability.

p. 399

III. *The Category of Possibility* [is not a constituate of Dewey's theory of the nature of mathematics or of its logical status. gp]

p. 401-2

"meaning" in mathematics

1. The relations of the map [of a country] are similar . . . to those of the country because both are *instituted by one and the same set of operations*, As far, then, as this case of similarity of relations is an illustration of isomorphism, it throws no light on the ontological isomorphism said to subsist in the case of mathematics. For that doctrine is at the opposite pole. It does not hod that operations that determine the relations of mathematical subject-matter also determine those of the "Realm of Possibilities." The position here taken does hold, however, that the operations of transformability which determine mathematical subject-matter are, or constitute the Realm of Possibilities in the only meaning logically assignable to that phrase.

relation of map to country

The statement that the relations of the map are similar to those of the country mapped because both are instituted by one and the same set of operations is readily seen by noting the fact that both are products of execution of certain operations that may be summed up in the word *surveying.* The elements of the country are certainly

existentially connected with one another. But as far as knowledge is concerned, as far as any propositions about these connections can be made, they are wholy indeterminate until the country is surveyed. When, and as far as, the country is surveyed, a map is brought into being. Then, of course, there is a common pattern of relations in the map and in the country as mapped. Any errors that result in the *map* from inadequacy in the operations of surveying will also be found in propositions about the relations of the *country.) The doctrine of structural (in the sense of non -operational* similarity of the relations of the map and those of the country is the product of taking maps that have in fact been perfected through performance of regulated operations of surveying in isolation from the operations by which the map was constructed. It illustrates the fallacy that always occurs when propositions are interpreted without reference to the means by which they are grounded.

2. Given the map as a pattern of relations, the "relation" of the pattern to that of the country mapped is functional. It is constituted through the intermediation of the *further* operations it directs – whose consequences, moreover, provide the means by which the validity of the map is tested. The map is instrumental to such operations as traveling, laying out routes for journeys, following movements of goods and persons. If this consideration is employed with respect to mathematical subject-matter, it must, of course, be noted that the further operations which the two respective subject-matters direct are of different forms. In the case of mathematics the operations and consequences are not existential as they are in the relation of the map to traveling, etc., and their consequences. But as far as *development* of mathematical subject-matter as such is concerned the analogy concerning the *functional* use of operations is precise. The reference of mathematical subject-matter that is given at any time is not ontological to a Realm of Possibilities, but to further operations of transformation.

p. 402-3

map

…. On the functional interpretation, any map in any system is "true" (that is, valid) if its operational use produces the consequences that are intended to be served by the map.[4]

p. 404

map validity

While it is not claimed that this operational-functional interpretation of isomorphic patterns of relationships, *disproves* the interpretation of mathematics that refers it to an ontological ground, it is claimed that it renders that interpretation unnecessary for *logical* theory, leaving it in the position of any metaphysical theory that must be argued for or against on *metaphysical* grounds.

p. 406

metaphysical grounding

The conceptual nature of the material data of mathematics means that they are determined exclusively and wholly in reference to the possibility of operations of transformation, the latter constituting procedural means. This property is all one with that freedom from specific and hence limiting interpretation …

freedom from interpretation

postulational method in mathematics

Discussion is thus brought to explicit consideration of the postulational method of mathematics. Any scientific system, when logically analyzed and ordered, is found to involve certain propositions that are, for that system, primitive. These primitive propositions are postulates in that they state *demands* to be satisfied by the derived propositions of the system. In the systems of natural science, the demands to be

[4]Interpretation of "truth" as correspondence in terms of literal reproduction would demand that a "true representation" be another globe just like the earth itself. Such a reproduction would be useless for the purpose representation fullfills. It would, in fact, only duplicate the problems of the original. (JD)

satisfied involve (1) elements determined by controlled or experimental observation and (2) operations which are capable of existential execution. The primitive propositions which are the postulates of a mathematical system are, as has been shown, free from both of these conditions. For their contents with respect to both elements and methods of operation are determined exclusively with reference to transformability.

The postulates of a mathematical system, in other words, state elements and ways of operating with them in strict conjugate relation each to the other ....

p. 407

The elements which are introduced by the postulates are specified in no other way than by the combinations into which they are permitted to enter by the postulates ....

.... The elements are what they are *defined* to be; constituted by definition and nothing but definition. The methods of operation, which are postulated in conjugate relation with the elements are, on the other hand, *resolutions* rather than definitions. Neither the definitions. Neither the definitions nor the resolutions can be identified with axioms in the traditional sense of self-evident truths. The resolution concerns methods of procedure to be strictly adhered to, and the definition posits elements to be operated with and upon by these specified methods of combination, yielding transformations stated in the theorems that follow. There is no other control of their meaning, which means that the control is strictly formal ....

postulates as demands

Every scientific system is constituted by a *set* of postulates, which in logical ideal are independent of one another, or that do not overlap as to operations to be performed. For a *combination* of operations is the only way in which development in discourse can take place.

p. 411

It is characteristic of the abstract universality of the transformability category in defining mathematical subject-matter that the institution of any given mathematical system sooner or later sets the problem of instituting a further branch of mathematics by means of which its characteristic theorems are translatable into those of other systems – a consideration that helps to explain the indefinite fertility of mathematical developments.

4.21 Scientific Method: Induction and Deduction

p. 419

4.22 Scientific Laws – Causation and Sequences

4.23 Scientific Method and Scientific Subject-Matter

p. 463

4.24 Social Inquiry

p. 487

4.25 The Logic of Inquiry and Philosophies of Knowledge

p. 513

References

[Pei31a] Charles Sanders Peirce. Collected Papers, Editors: Charles Hartshorne and Paul Weiss. Harvard University Press, Cambridge (MA), 1931. Volumes 1-6, 1931-35. $^{2}1960$: The Belknap Press of Harvard Univeristy Press. 4.25

[Pei34] Charles Sanders Peirce. *Pragmatism and Pragmaticism*. Volume 5 of *Collected Papers, Editors: Charles Hartshorne and Paul Weiss* [Pei31a], 1934. $^{2}1960$, $^{3}1965$. 6

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